

PID Controller Tuning for Desired Closed-Loop Responses for SI/SO Systems

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Proportional, integral, and derivative (PID) parameters are obtained for general process models by approximating the feedback form of an IMC controller with a Maclaurin series in the Laplace variable. These PID parameters yield closed-loop responses that are closer to the desired responses than those obtained by PID controllers tuned by other methods. The improvement in closed-loop control performance becomes more prominent as the dead time of the process model increases. A new design method for two degree of freedom controllers is also proposed. Such controllers are essential for unstable processes and provide significantly improved dynamic performance over single degree of freedom controllers for stable processes when the disturbances enter through the process.

Introduction

Since the proportional, integral, and derivative (PID) controller finds widespread use in the process industries, a great deal of effort has been directed at finding the best choices for the controller gain, integral, and derivative time constants for various process models (Ziegler and Nichols, 1942; Cohen and Coon, 1953; Lopez et al., 1967; Smith et al., 1975; Rivera et al., 1986; Chien and Fruehauf, 1990; Tyreus and Luyben, 1992; Sung et al., 1995; Lee et al., 1996). Among the performance criteria used for PID controller parameter tuning, the criterion to keep the controlled variable response close to the desired closed-loop response has gained widespread acceptance in the chemical process industries because of its simplicity, robustness, and successful practical applications. The IMC (internal model control)-PID tuning method (Rivera et al., 1986; Morari and Zafiriou, 1989) and the direct synthesis method (Smith et al., 1975) are typical of the tuning methods based on achieving a desired loop response. They obtain the PID controller parameters by first computing the controller which gives the desired closed-loop response. Generally, this controller is rather more complicated than a PID

controller. However, by clever approximations of the process model, the controller form can be reduced to that of a PID controller, or a PID controller cascaded with a first- or second-order lag. An important advantage of such methods is that the closed-loop time constant, which is the same as the IMC filter time constant, provides a convenient tuning parameter to adjust the speed and robustness of the closed-loop system. Intuitively, one would expect that as the desired closed-loop time constant increases, the PID controller gain and derivative time constants would decrease. The PID controller gain does indeed behave as expected. However, most tuning methods yield derivative and integral time constants that are independent of the closed-loop time constant. Also, current tuning methods yield PID parameters only for a restricted class of process models. There is no general methodology for arbitrary process models other than approximating them with a first- or second-order models and applying tuning rules for the approximate models.

In this article, we generalize the IMC-PID approach and obtain the PID parameters for general models by approximating the ideal controller with a Maclaurin series in the Laplace variable. It turns out that the PID parameters so obtained provide somewhat better closed-loop responses than

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those obtained previously. Further, all of the PID parameters depend on the desired closed-loop time constant in a manner consistent with engineering intuition. Several examples are provided to demonstrate the method and to compare results with alternate tuning methods.

Development of General Tuning Algorithm for PID Controllers

Single degree of freedom controllers (q_r and $G_D = 1$ in Figure 1)

Consider stable (that is, no right half plane poles) process models of the form

$$G(s) = p_m(s)p_A(s) \quad (1)$$

where $p_m(s)$ is the portion of the model inverted by the controller (it must be minimum phase), $p_A(s)$ is the portion of the model not inverted by the controller (it is usually nonminimum phase, that is, it contains dead times and/or right half plane zeros) and $p_A(0) = 1$.

Often, the portion of the model not inverted by the controller is chosen to be all pass (that is, of the form

$$p_A(s) = \prod_{i,j} \left(\frac{-\tau_i s + 1}{\tau_i s + 1} \right) \left(\frac{\tau_j^2 s^2 - 2\tau_j \zeta_j s + 1}{\tau_j^2 s^2 + 2\tau_j \zeta_j s + 1} \right) e^{-T_s}$$

$$\tau_i, \tau_j > 0; 0 < \zeta_j < 1$$

since this choice gives the best least-squares response. The requirement that $p_A(0) = 1$ is necessary for the controlled variable to track its set point.

Our aim is to choose the controller G_c of Figure 1 to give the desired closed-loop response, C/R of

$$\frac{C}{R} = \frac{p_A(s)}{(\lambda s + 1)^r} \quad (2)$$

The term $1/(\lambda s + 1)^r$ functions as a filter with an adjustable time constant λ , and an order r chosen so that the controller G_c is realizable.

The ideal controller G_c that yields the desired loop response given by Eq. 2 perfectly is given by

$$G_c(s) = \frac{q}{(1 - Gq)} = \frac{p_m^{-1}(s)}{(\lambda s + 1)^r - p_A(s)} \quad (3)$$

where q is the IMC controller

$$p_m^{-1}(s)/(\lambda s + 1)^r$$

Although the resulting controller is physically realizable, it does not have the standard PID form. Therefore, the main issue for developing the tuning rule to give the desired closed-loop response is to find the PID parameters that approximate the response of the ideal controller given by Eq. 3. One approach is to force the controller transfer function given by Eq. 3 into the standard PID form by approximating the

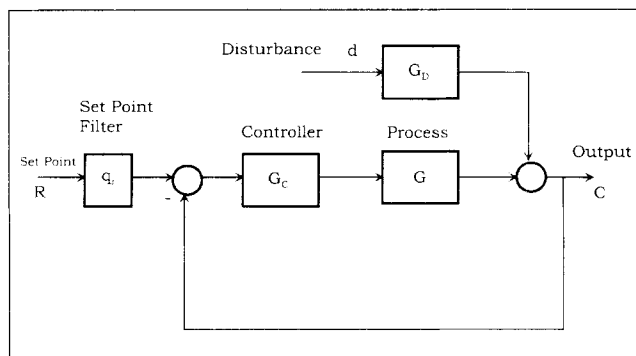


Figure 1. Feedback control system.

dead time with a low-order Padé approximation. The tuning rules given by Smith et al. (1975), Rivera et al. (1986), and Morari and Zafiriou (1989) are based on this approach. We propose a new approach to obtain the PID controller that approximates the ideal controller given by Eq. 3 more closely.

The controller G_c can be approximated by a PID controller by first noting that it can be expressed as

$$G_c \equiv f(s)/s \quad (4)$$

Whereas G_c has a pole at the origin because $p_A(0)$ is one, $f(s)$ will not have such a pole because the derivative of $((\lambda s + 1)^r - p_A(s))/s$ at the origin is never zero for r greater than zero.

Expanding $G_c(s)$ in a Maclaurin series in s gives

$$G_c(s) = \frac{1}{s} \left[f(0) + f'(0)s + \frac{f''(0)}{2}s^2 + \dots \right] \quad (5)$$

It should be noted that the resulting controller has the proportional term, integral term and derivative term, in addition to an infinite number of higher-order derivative terms. Since the controller given by Eq. 5 is equivalent to the ideal controller given by Eq. 3, the desired closed-loop response can be perfectly achieved if all terms in Eq. 5 are implemented. In practice, however, it is impossible to implement the controller given by Eq. 5 because of the infinite number of high-order derivative terms. In fact, in an actual control situation low and middle frequencies are much more important than high frequencies, and only the first three terms in Eq. 5 are often sufficient to achieve the desired closed-loop performance. The controller given by Eq. 5 can be approximated to the PID controller by using only the first three terms ($1/s$, 1 , s) in Eq. 5 and truncating all other high-order terms (s^2 , s^3 , ...). The first three terms of the above expansion can be interpreted as the standard PID controller given by

$$G_c(s) = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s \right) \quad (6)$$

where

$$K_c = f'(0) \quad (7a)$$

$$\tau_I = f'(0)/f(0) \quad (7b)$$

$$\tau_D = f''(0)/2f'(0) \quad (7c)$$

In order to evaluate the PID controller parameters given by Eqs. 7a–7c, we let

$$D(s) \equiv ((\lambda s + 1)^r - p_A(s))/s \quad (8a)$$

Then, by Maclaurin series expansion we get

$$D(0) = r\lambda - p'_A(0) \quad (8b)$$

$$D'(0) = [r(r-1)\lambda^2 - p''_A(0)]/2 \quad (8c)$$

$$D''(0) = [r(r-1)(r-2)\lambda^3 - p'''_A(0)]/3 \quad (8d)$$

Using Eq. 8, the function $f(s)$ and its first and second derivatives, all evaluated at the origin, are given by

$$f(0) = \frac{1}{K_p D(0)} \quad (9a)$$

$$f'(0) = \frac{-[p'_m(0)D(0) + K_p D'(0)]}{[K_p D(0)]^2} \quad (9b)$$

$$f''(0) = f'(0)$$

$$\left[\left(\frac{p''_m(0)D(0) + 2p'_m(0)D'(0) + K_p D''(0)}{p'_m(0)D(0) + K_p D'(0)} \right) + 2f'(0)/f(0) \right] \quad (9c)$$

where

$$K_p \equiv p_m(0) = G(0)$$

The above formulas can be used to obtain the controller gain, and integral and derivative time constants as analytical functions of the process model parameters and the closed-loop time constant λ , as is done in the next section for several examples.

The derivative and/or integral time constants computed from Eq. 7 can be negative for some process models independent of the choice of the closed-loop time constant λ . When this occurs, it is because a simple PID controller cannot achieve the desired critically damped closed-loop behavior. In this case the designer has at least two alternatives. One alternative is to modify the desired closed-loop behavior from critically damped to underdamped by choosing an underdamped filter. The designer must then seek a damping ratio for the filter that results in positive integral and derivative PID parameters. Rather than such an approach, we recommend replacing the simple PID controller with a PID controller cascaded with a first- or second-order lag of the form $1/(\alpha s + 1)$ or $1/(\beta_2 s^2 + \beta_1 s + 1)$, respectively. To obtain a PID controller cascaded by a first-order lag [that is, $K_c(1 + 1/\tau_I s + \tau_D s)/(\alpha s + 1)$], we rewrite $G_c(s)$ as

$$G_c(s) \equiv \frac{1}{2} f(s) = \frac{1}{2} \frac{[f(s)h(s)]}{h(s)} \quad (10)$$

where

$$h(s) \equiv 1 + \alpha s$$

Now, we expand the quantity $f(s)h(s)$ in a Maclaurin series about the origin and choose the parameter α so that the third-order term in the expansion becomes zero.

The expansion of Eq. 10 then becomes

$$G_c(s) = \{f(0) + [f'(0) + \alpha f(0)]s + [f''(0) + 2\alpha f'(0)]s^2/2 + [f'''(0) + 3\alpha f''(0)]s^3/6 + \dots\}/s(\alpha s + 1) \quad (11)$$

Selecting the lag parameter α to drop the third-order term gives

$$\alpha = -f'''(0)/3f''(0) \quad (12a)$$

and the PID parameters are

$$K_c = f'(0) + \alpha f(0); \tau_I = K_c/f(0);$$

$$\tau_D = [f''(0) + 2\alpha f'(0)]/(2K_c) \quad (12b)$$

Again, the PID-lag controller is

$$K_c(1 + 1/\tau_I s + \tau_D s)/(\alpha s + 1) \quad (12c)$$

To obtain a PID controller cascaded with a second-order lag, we write $G_c(s)$ from Eq. 3 as

$$G_c(s) \equiv \frac{N(s)}{sD(s)} \quad (13)$$

where $N(s)$ and $D(s)$ are polynomials obtained by substituting high-order (≥ 4) Padé approximations for the exponential terms in $p_m(s)$ and $p_A(s)$. This gives

$$G_c(s) = \frac{k(\lambda) \left(\sum_{i=1}^n \alpha_i s^i + 1 \right)}{s \left(\sum_{i=1}^{n-1} \beta_i(\lambda) s^i + 1 \right)} \quad (14)$$

where $\alpha_i, \beta_i(\lambda) \geq 0$ and $\beta_i(\lambda), k(\lambda)$ are functions of λ .

Dropping terms higher than second-order in the numerator and higher than third order in the denominator gives

$$G_c(s) \equiv \frac{k(\lambda)}{s} \frac{(\alpha_2 s^2 + \alpha_1 s + 1)}{[\beta_2(\lambda) s^2 + \beta_1(\lambda) s + 1]} \quad (15)$$

The controller given by Eq. 15 can be viewed as an ideal PID controller cascaded with a second-order lag or as a floating integral controller cascaded with a second-order lead-lag transfer function. The controller parameters are $K_c = k(\lambda) \cdot \tau_I$; $\tau_I = \alpha_1$; $\tau_D = \alpha_2/\tau_I$.

The second-order lag is given by $(1/(\beta_2 s^2 + \beta_1 s + 1))$. All of the parameters except possibly K_c are positive.

It is relatively easy to write a macro for Program CC (Commercial software for control system design and analysis, which is available from Systems Technology Inc. in Hawthorne, CA) to compute the parameters for the three PID controllers given by Eqs. 7, 12 and 15. One such macro, which we call PIDIMC,

is an ASCII file that can be obtained either via the Internet at <http://k2.scl.cwru.edu/cse/eche/faculty/brosilow.htm> or by anonymous ftp to [cheme.cwru.edu](ftp://cheme.cwru.edu) (You must change directory (cd) to `pub/process_control/ProgramCC`). Input parameters for the macro are: the name of the part of the model that the controller inverts, the name of the part of the model (exclusive of the dead time) that the controller does not invert, the dead time, the order of the closed-loop lag r [$Pm(s)$], the value of the lag time constant λ , and, finally, a five-letter identifier (optional). The macro also computes and displays the closed-loop responses for each of the three types of PID controller as well as the IMC response.

The same calculations are also performed by a MATLAB program that we call IMCTUNE. This program can also be obtained at the Internet address given above, as well as by anonymous ftp to [cheme.cwru.edu/pub/process_control/IMCTUNE](ftp://cheme.cwru.edu/pub/process_control/IMCTUNE).

Two degree of freedom controllers

Two degree of freedom controllers are useful when the lag in G_D of Figure 1 is on the order of, or larger than, the process lag in G . Such controllers also provide significantly improved dynamic performance over single degree of freedom controllers when the process is unstable and disturbances enter through the process (that is, G_D contains the same unstable lags as the process). The design of two degree of freedom controllers proceeds by specifying the desired closed-loop set point response and an approximate disturbance response. This is most easily accomplished by selecting the feedback controller G_c as

$$G_c = qq_d / (1 - Gqq_d) \quad (16)$$

Notice, that the form of G_c in Eq. 16 is similar to that in Eq. 3, the difference being that the IMC controller q in Eq. 3 has been replaced by two controllers q and q_d .

With the above choice for the controller G_c , the closed-loop set point and disturbance responses become

$$C(s)/R(s) = q_r q_d G q \quad (17)$$

$$C(s)/d(s) = (1 - Gqq_d) G_D \quad (18)$$

If we choose q_r as the inverse of q_d , and if we select q as in Eq. 3, then the set point response is the same as that given by Eq. 2 for the single degree of freedom controller.

To shape the disturbance response given by Eq. 18, common practice is to select q_d so that the zeros of $(1 - Gqq_d)$ cancel the large, or unstable, time constants in G_D . A convenient form for q_d is

$$q_d = \left(\sum_{i=1}^m \alpha_i s^i + 1 \right) / (\lambda s + 1)^m \quad (19)$$

The order, m , of q_d is equal to the number of poles of G_D to be canceled by the zeros of $(1 - Gqq_d)$. Usually, m is on the order of one or two. The constants α_i are chosen to cancel the desired poles in G_D .

Once the closed-loop time constant λ has been selected,

and the α_i calculated, then G_c from Eq. 16 can be expanded in a Maclaurin series, just as was done for the single degree of freedom controller. The only difference is that care must be taken to cancel common factors in the numerator and denominator of G_c before the expansion. Common factors always occur when G contains one or more of the poles of G_D , which are removed by the zeros of $(1 - Gqq_d)$. In this case, the IMC controller q has common zeros with $(1 - Gqq_d)$, and these common factors must be removed before expansion.

One final caution in the design of two degree of freedom controllers is that the suggested selection procedure for the parameters α_i can lead the term $(1 - Gqq_d)$ to have right half plane zeros which are not canceled by corresponding zeros in q . In this case, the controller G_c is unstable, and should generally not be implemented as such. Usually, the foregoing problem can be overcome at the price of increasing the closed-loop time constant λ . An alternative is to insert the offending zeros into the IMC controller q , thereby introducing nonminimum phase behavior in the closed-loop set point response.

In order to achieve a desirable set point response, especially in the presence of modeling error, it is often desirable to select a filter time constant λ_r for the set point filter q_r , which is different from the controller filter time constant λ . The form of the set point filter then becomes

$$q_r(\lambda, \lambda_r) = q(s, \lambda_r) / qq_d(s, \lambda) \quad (20)$$

Substituting Eq. 20 into 17 shows that the set point response of the two degree of freedom control system for a perfect model is the same as that of a single degree of freedom control system with a filter time constant of λ_r .

The software IMCTUNE, discussed previously, calculates the parameters of two degree of freedom PID controllers as well as the filter q_r for $m = 1$ or 2. The Program CC macro entitled 2DFPID performs the same calculations, but is limited to $m = 1$. This macro can be obtained as described for the macro PIDIMC. In both programs the only additional data needed is the lag in G_D that is to be canceled by the zeros of $(1 - Gqq_d)$.

Examples

One degree of freedom controllers

First-Order Plus Dead Time (FOPDT) Model. The most commonly used approximate model for chemical processes is the first-order plus dead time model given below

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \quad (21)$$

Specifying a desired closed-loop response of the form $C/R = e^{-\theta s} / (\lambda s + 1)$ evaluating the PID parameters from Eqs. 7, 8 and 9 gives

$$K_c = \frac{\tau_I}{K(\lambda + \theta)}, \quad \tau_I = \tau + \frac{\theta^2}{2(\lambda + \theta)}, \quad \tau_D = \frac{\theta^2}{2(\lambda + \theta)} \left[1 - \frac{\theta}{3\tau_I} \right] \quad (22)$$

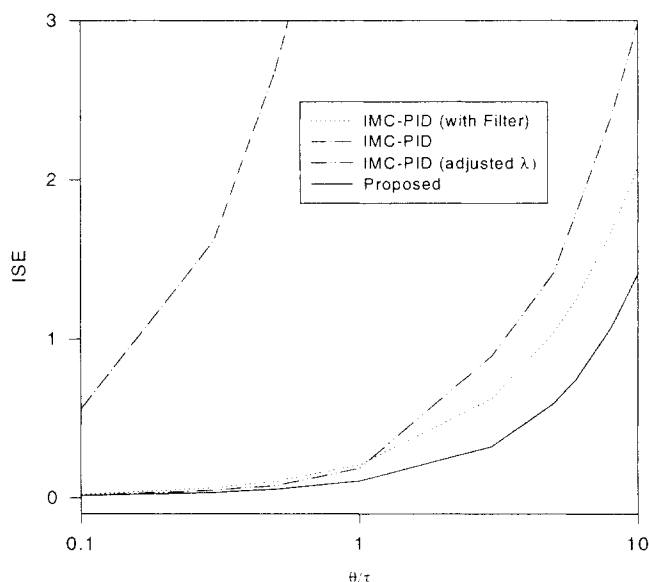


Figure 2. Comparison of the ISE generated by various tuning rules.

Notice that as the desired loop time constant λ gets large, the controller integral time constant τ_I , approaches the process model time constant τ , and the controller gain K_c and derivative time constant both approach zero. Thus the PID controller goes smoothly into a PI controller, and then a floating integral controller, as the desired speed of response decreases.

Figure 2 compares the integral of the squared error for step set point changes using the tuning rule given by Eq. 22 with those given by Rivera et al. (1986) for varying process dead time to time constant ratios and λ chosen as $\lambda/\theta = 1/3$. To obtain a fair comparison, the same value of λ was used

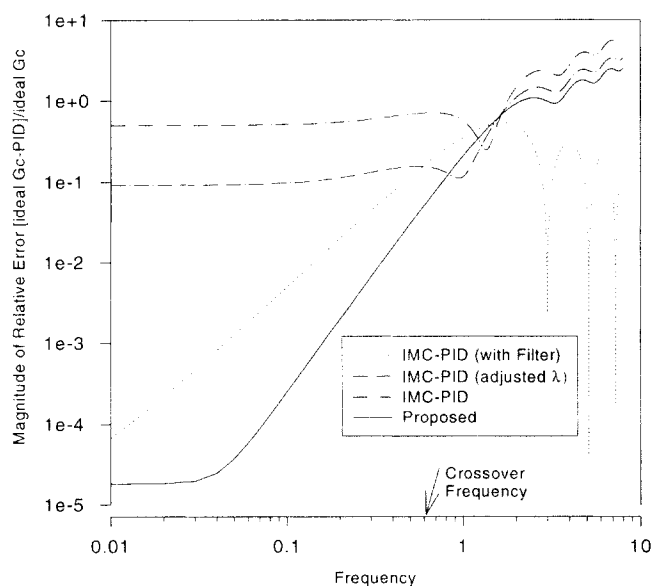
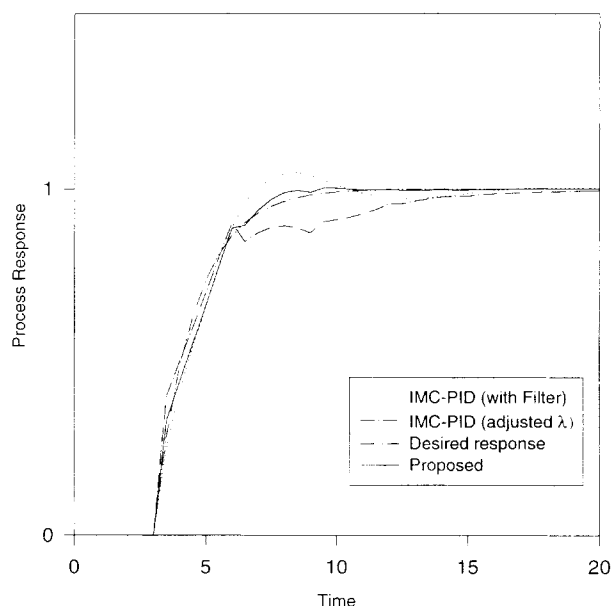
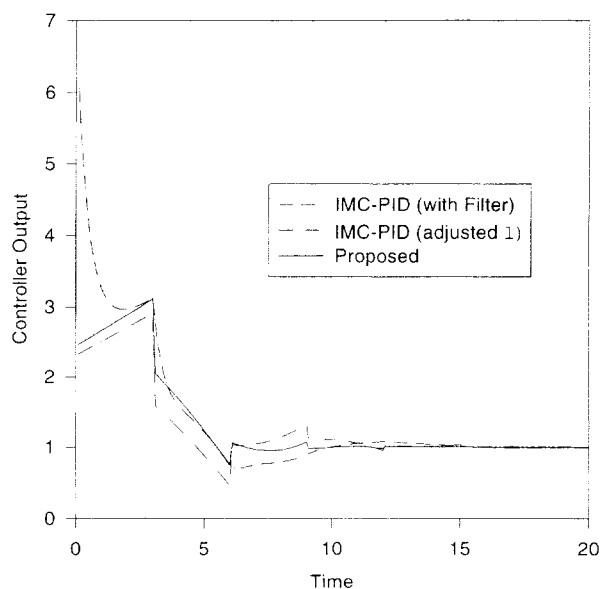


Figure 3. Relative errors of the PID controllers by various tuning rule to the ideal controller for $G = e^{-3s}/(10s+1)$; $\lambda = 1.5$; $\lambda_{\text{adjusted}} = 3.48$.

for each tuning rule. It is not surprising that the IMC-PID method with no adjustment on the λ value shows the largest ISE. This result is due to the fact that λ in the IMC-PID tuning rule is not equivalent to the closed-loop time constant. To improve the IMC-PID response, we computed an adjusted λ , which gives the minimum ISE for the response specified by the particular closed-loop time constant by solving the associated nonlinear optimization problem. The result is denoted as IMC-PID (adjusted λ) in Figure 2. As seen from the figure, the proposed tuning rule gives the smallest ISE among all tuning rules over the entire range of θ/τ . The difference in the values of the ISE becomes more significant as the dead time effect dominates. The proposed tuning is



(a) Closed-loop responses;



(b) controller outputs.

Figure 4. Responses to a unit step change in set point for $G(s) = e^{-3s}/(10s+1)$; $\lambda = 1.5$; $\lambda_{\text{adjusted}} = 3.48$.

superior throughout. As an alternate comparison of the approximation precision of the PID controllers based on different tuning rules, the norms of the relative errors between the PID controllers and the ideal controller given by Eq. 3 vs. frequency are plotted in Figure 3. The process $G = e^{-3s}/(10s + 1)$ was used in the simulation. As can be seen in the figure, the proposed tuning rule shows relatively small error for the low and middle frequency range. Note that in this example the cross-over frequency that is important in control is 0.61. In the high frequency range, the IMC-PID (with filter) method yields smaller relative error over the proposed methods because of the filter.

Figures 4a and b show the closed-loop response and controller output respectively for dead time to time constant ratios of 0.3. The values of the controller parameters used in Figure 4 are as follows

IMC-PID with filter ($K_c = 2.555$, $\tau_I = 11.5$, $\tau_D = 1.304$,
 $\tau_F = 0.5$)

IMC-PID (adjusted) ($K_c = 2.309$, $\tau_I = 11.5$, $\tau_D = 1.304$)

Proposed Controller ($K_c = 2.444$, $\tau_I = 11$, $\tau_D = 0.909$)

Second-Order Plus Dead Time (SOPDT) Model. For a process of the form given by Eq. 23 below, and a desired closed-loop response of $C/R = e^{-\theta s}/(\lambda s + 1)^2$, evaluating Eqs. 7, 8, and 9 gives the PID parameters shown in Eq. 24 below (after some tedious algebra)

$$G(s) = \frac{Ke^{-\theta s}}{(\tau^2 s^2 + 2\zeta\tau s + 1)}, \tau, \zeta, > 0 \quad (23)$$

$$K_c = \tau_I / [K(2\lambda + \theta)] \quad (24a)$$

$$\tau_I = 2\zeta\tau - (2\lambda^2 - \theta^2) / [2(2\lambda + \theta)] \quad (24b)$$

$$\tau_D = \tau_I - 2\zeta\tau + [\tau^2 - \theta^3 / 6(2\lambda + \theta)] / \tau_I \quad (24c)$$

For process models of the form $Ke^{-\theta s}/[(\tau_1 s + 1)(\tau_2 s + 1)]$, simply replace $2\zeta\tau$ and τ^2 in Eq. 24 with $\tau_1 + \tau_2$ and $\tau_1\tau_2$, respectively. For comparison, the tuning rule of Smith et al. (1975) is also shown in Table 1.

Figure 5 compares the closed-loop responses by several tuning methods for the process given by Eq. 24 with $\tau = 10$ and $\zeta = 1$. The resulting PID controller by the proposed method performs better than the controller tuned by the Smith method. Figure 6 compares the performance of various tuning rules using an approximate model. The process $G = e^{-10s}/(10s + 1)^2$ is approximated by a model $G \approx e^{-15.5s}/(15s + 1)$. The superior performance of the proposed method is readily apparent from the figure. The tuning rules given by Eqs. 24a to 24c produce a closed-loop response of the form $e^{-\theta s}/(\lambda s + 1)^2$ for a perfect model. However, it is also possible to get a response that approximates $e^{-\theta s}/(\lambda s + 1)$, simply by choosing a first-order lag for the IMC controller q . The fact that such an IMC controller isn't realizable doesn't matter since only the PID controller is desired. The tuning rules for the first-order lag plus dead time response are

$$K_c = \frac{\tau_I}{K(\lambda + \theta)}, \quad \tau_I = 2\zeta\tau + \frac{\theta^2}{2(\lambda + \theta)},$$

$$\tau_D = \tau_I - 2\zeta\tau - \frac{\tau^2 - \frac{\theta^3}{6(\lambda + \theta)}}{\tau_I} \quad (24d)$$

Table 1. Various Tuning Rules to Give the Desired Closed-Loop Response*

Process Model	Tuning Method	K_c	τ_I	τ_D	τ_F
$G = \frac{Ke^{-\theta s}}{\tau s + 1}$	Rivera et al.	$\frac{1}{K} \frac{2\tau + \theta}{2(\lambda + \theta)}$	$\tau + \frac{\theta}{2}$	$\frac{\tau\theta}{2\tau + \theta}$	
	Rivera et al. (with Filter)	$\frac{2\tau + \theta}{2K(\lambda + \theta)}$	$\tau + \frac{\theta}{2}$	$\frac{\tau\theta}{2\tau + \theta}$	$\frac{\lambda\theta}{2(\lambda + \theta)}$
	Proposed	$\frac{\tau_I}{K(\lambda + \theta)}$	$\tau + \frac{\theta^2}{2(\lambda + \theta)}$	$\frac{\theta^2}{6(\lambda + \theta)} \left[3 - \frac{\theta}{\tau_I} \right]$	
$G = \frac{Ke^{-\theta s}}{\tau s + 1}$	Smith	$\frac{\tau}{K(\lambda + \theta)}$	τ		
	Rivera et al. Improved IMC-PI	$\frac{2\tau + \theta}{2K\lambda}$	$\tau + \frac{\theta}{2}$		
	Proposed	$\frac{\tau_I}{K(\lambda + \theta)}$	$\tau + \frac{\theta^2}{2(\lambda + \theta)}$		
$G = \frac{Ke^{-\theta s}}{(\tau^2 s^2 + 2\zeta\tau s + 1)}$	Smith	$\frac{\tau_1 + \tau_2}{K(\lambda + \theta)}$	$\tau_1 + \tau_2$	$\frac{\tau_1\tau_2}{\tau_1 + \tau_2}$	
	Proposed	$\frac{\tau_I}{K(2\lambda + \theta)}$	$2\zeta\tau - \frac{2\lambda^2 - \theta^2}{2(2\lambda + \theta)}$	$\tau_I - 2\zeta\tau + \frac{\tau^2 - \frac{\theta^3}{6(2\lambda + \theta)}}{\tau_I}$	

*Desired closed-loop response: $C/R = e^{-\theta s}/(\lambda s + 1)$, $r = 1$ or 2

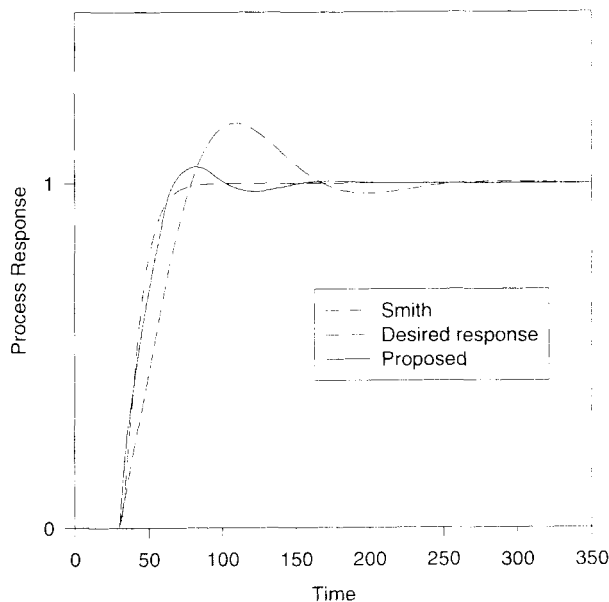


Figure 5. Closed-loop responses to a unit step change in set point: $G(s) = e^{-30s}/[(10s+1)(10s+1)]$; $\lambda = 7$ (proposed) $\lambda = 15$ (Smith).

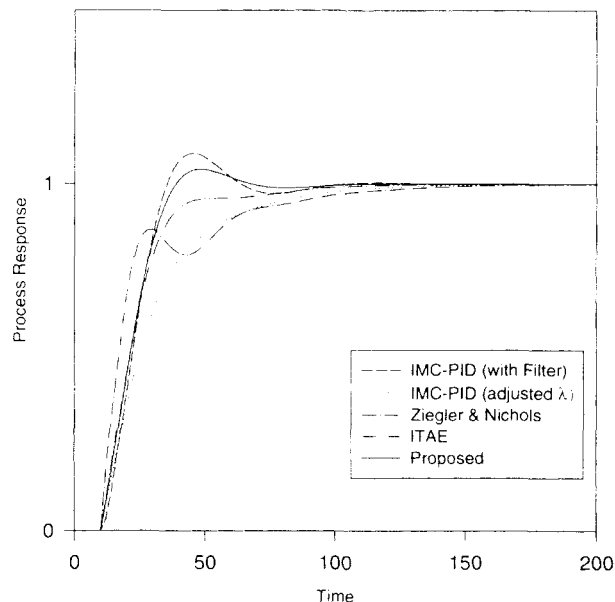


Figure 6. Closed-loop responses to a unit step change in set point: $G = e^{-10s}/(10s+1)$; $\lambda = 6$; $\lambda_{\text{adjusted}} = 24$.

Notice that Eqs. 24a to 24c are almost the same as Eq. 24d if λ in Eq. 24d is taken as being twice that used in Eqs. 24a to 24c. The resulting step set point responses are also almost the same.

As stated previously, the derivative and/or integral time constants computed from Eqs. 7, 8 and 9 can be negative for some process models independent of the choice of filter time constant. This often occurs when the process model has one or more dominant lead time constants as for example the process given by

$$\tilde{p}(s) = \frac{s^2 + 2s + 0.25}{s^4 + 6.5s^3 + 15s^2 + 14s + 4} \quad (25a)$$

$$= \frac{0.0625(7.46s + 1)(0.536s + 1)}{(2s + 1)(0.5s + 1)^3} \quad (25b)$$

The open-loop response of the above process to a unit step change in control effort is given in Figure 7. Notice the very large overshoot of the final steady state caused by the strong lead action of the term $(7.46s + 1)$ in the numerator of Eq. 25b. Using Eqs. 7, 8, and 9 to compute PID parameters with a filter time constant of .2 yields a PID controller with $\tau_I = -4.60$ and $\tau_D = -7.87$. On the other hand, using Eqs. 12, 8 and 9 gives

$$\text{PID} \cdot \text{lag} = \frac{40(1.19s^2 + 2.86s + 1)}{s(7.47s + 1)} \quad (26)$$

The response of the closed-loop control system using the above PID · lag controller is given in Figure 8. Notice that the lag time constant in Eq. 26 is nearly the same as the large lead time constant in the process model. Indeed, very nearly the same controller would have been obtained by finding the PID controller for the process given by Eq. 26b, but with the

lead removed. However, to obtain this controller it is necessary to use a second-order filter with a time constant of 0.2 to make the controller proper.

The PID controller with a second-order lag seems most useful when the process model has a strong second-order lead with complex zeros. For example, consider the process given by

$$\tilde{p}(s) = \frac{0.5(16s^2 + 0.4s + 1)}{(2s + 1)(0.5s + 1)^3} \quad (27)$$

Using a filter time constant of 0.5 yields an integral time con-

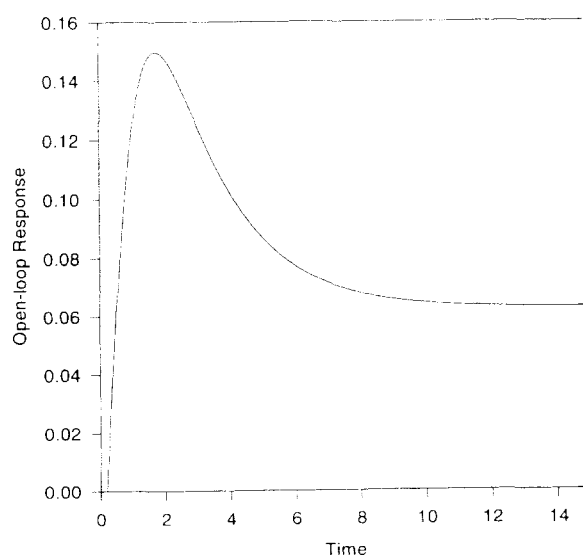


Figure 7. Step response: $G(s) = (s^2 + 2s + 0.25)/(s^4 + 6.5s^3 + 15s^2 + 14s + 4)$.

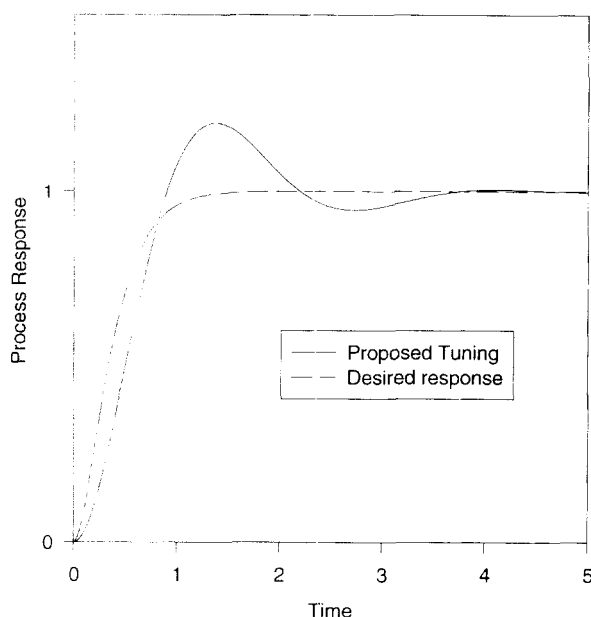
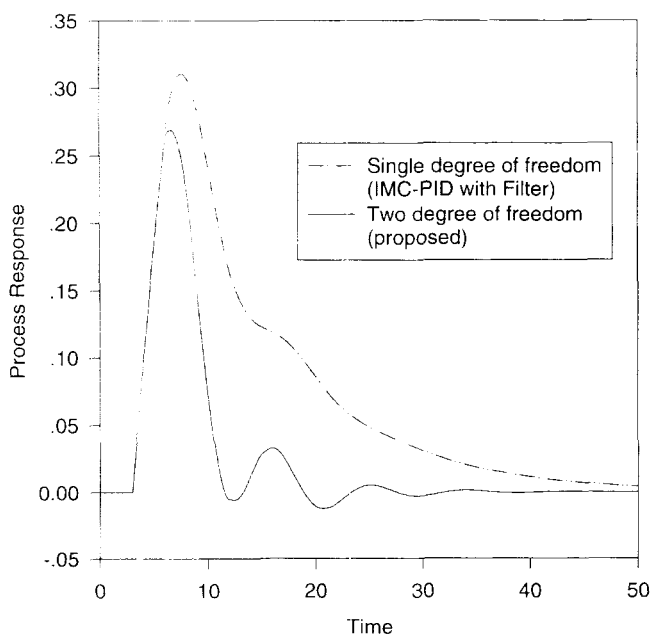


Figure 8. Closed-loop response: $G(s) = (s^2 + 2s + 0.25) / (s^4 + 6.5s^3 + 15s^2 + 14s + 4)$ with the PID-lag controller $[40(1.19s^2 + 2.86s + 1)] / [s(7.47s + 1)]$.

stant of 2.85 and a derivative time constant of -4.98 . The lag time constant computed from Eq. 12a is -2.75 , and so the controller given by Eq. 12c also cannot be used. Finally, the controller given by Eq. 15 for a filter time constant of 0.5 is

$$\text{PID} \cdot \text{lag} = \frac{2(3.75s^2 + 3.5s + 1)}{s(16.1s^2 + 0.65s + 1)} \quad (28)$$



(a) Disturbance rejection responses;

Notice that the denominator lag of Eq. 28 is very close to the numerator lead in the process model given by Eq. 27. Here again, if one eliminates the numerator term $(16s^2 + 4s + 1)$ from the process model, computes the PID controller for the reduced model, and then adds back into the controller a lag to cancel the numerator term, the result is

$$\text{PID} \cdot \text{lag} = \frac{2(2.94s^2 + 3.25s + 1)}{s(16s^2 + 4s + 1)} \quad (29)$$

The control system response using Eq. 29 is very similar to that using Eq. 28, and both yield excellent approximations to the desired closed-loop response of $1/(0.5s + 1)^2$.

Two degree of freedom controllers

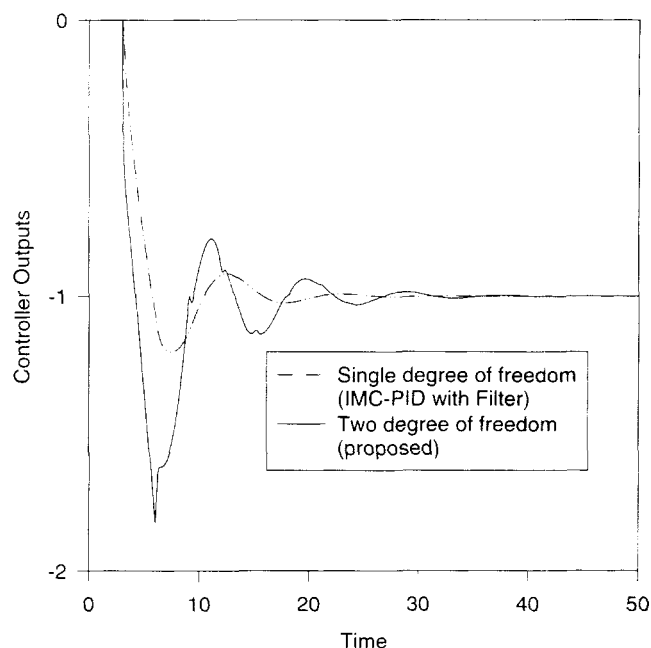
FOPDT Model. For a process of the form given by Eq. 30 below, specifying a desired closed-loop response of the form $C/R = e^{-\theta s} / (\lambda s + 1)$ and q_d of the form $q_d(s) = (\alpha s + 1) / (\lambda s + 1)$ gives the controller shown in Eq. 31

$$G(s) = G_D(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \quad (30)$$

$$G_c = \frac{qq_d(s)}{1 - G(s)qq_d(s)} = \frac{(\tau s + 1)(\alpha s + 1)}{K[(\lambda s + 1)^2 - (\alpha s + 1)e^{-\theta s}]} \quad (31)$$

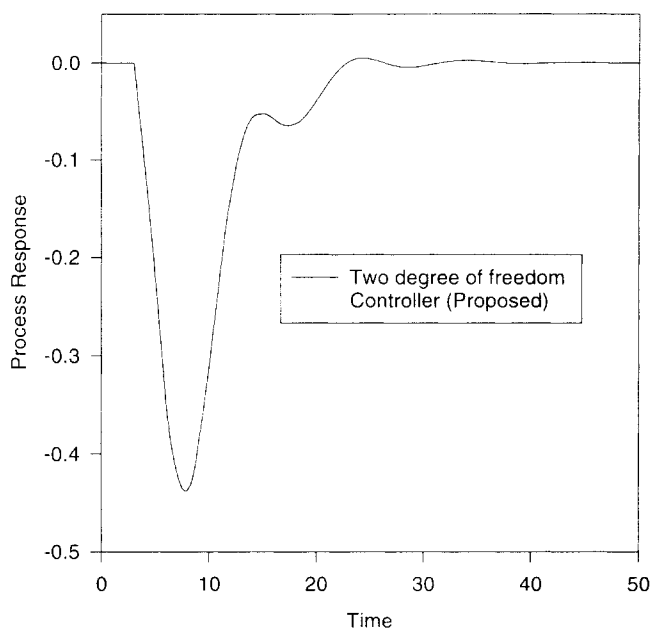
Evaluating the PID parameters from Eqs. 7, 8 and 9 gives

$$K_c = \frac{\tau_I}{K(2\lambda + \theta - \alpha)} \quad (32a)$$

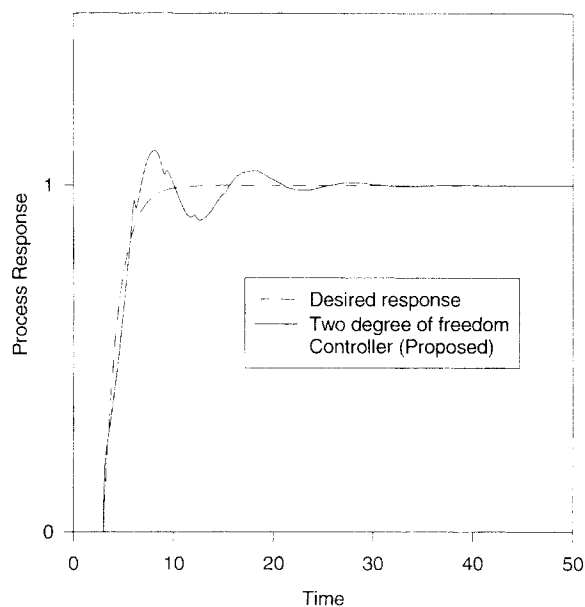


(b) controller outputs.

Figure 9. Comparison of single degree of freedom controller with $\lambda = 0.75$ and two degree of freedom controller with $\lambda = \lambda_r = 1.5$ for $G(s) = e^{-3s} / (10s + 1)$.



(a) Disturbance rejection;



(b) set point tracking.

Figure 10. Closed-loop responses by two degree of freedom controller with $\lambda=2.5$ and $\lambda_r=1.5$ for the unstable process $G(s)=e^{-3s}/(-10s+1)$.

$$\tau_I = \tau + \alpha - \frac{\lambda^2 + \alpha\theta - \theta^2/2}{2\lambda + \theta - \alpha} \quad (32b)$$

$$\tau_D = \frac{\tau\alpha - \frac{\theta^3/6 - \alpha\theta^2/2}{2\lambda + \theta - \alpha}}{\tau_I} - \frac{\lambda^2 + \alpha\theta - \theta^2/2}{2\lambda + \theta - \alpha} \quad (32c)$$

Here, we want to choose α so that the term $[1 - G(s)qq_d(s)]$ has a zero at the pole of $G_D(s)$. That is, we want

$$[1 - G(s)qq_d(s)]|_{s=-1/\tau} = 0 \quad (33)$$

$$\left[1 - \frac{(\alpha s + 1)}{(\lambda s + 1)^2} e^{-\theta s} \right] \Big|_{s=-1/\tau} = 0 \quad (34)$$

The solution of Eq. 34 gives the parameter α as a function of the filter and model time constants λ and τ . Figure 9a compares the closed-loop disturbance rejections by single degree of freedom controller and two degree of freedom controller for the process given by Eq. 30 with $K=1$, $\tau=10$, and $\theta=3$. Figure 9b shows controller outputs for the responses in Figure 9a. The two degree of freedom controller gives better performance than the single-degree of freedom controller.

Figures 10a and 10b show the closed-loop disturbance and set point responses of a two-degree of freedom controller for the unstable process given by Eq. 30 with $K=1$, $\tau=-10$, and $\theta=3$. Disturbances enter through the process [that is, $G_D(s)=1/(-10s+1)$]. These parameters yield the following controller: $K_C = -4.018$, $\tau_I = 12.42$, $\tau_D = 1.217$

$$q_r = (6.25s^2 + 5s + 1)/(16.64s^2 + 12.59s + 1)$$

SOPDT Model. For a process of the form given by Eq. 35 below, specifying a desired closed-loop response of the form $C/R = e^{-\theta s}/(\lambda s + 1)^2$ and q_d of the form $q_d(s) = (\alpha_2 s^2 + \alpha_1 s + 1)/(\lambda s + 1)^2$ gives the controller shown in Eq. 36.

$$G(s) = G_D = \frac{K_e^{-\theta s}}{(\tau^2 s^2 + 2\zeta\tau s + 1)} \quad (35)$$

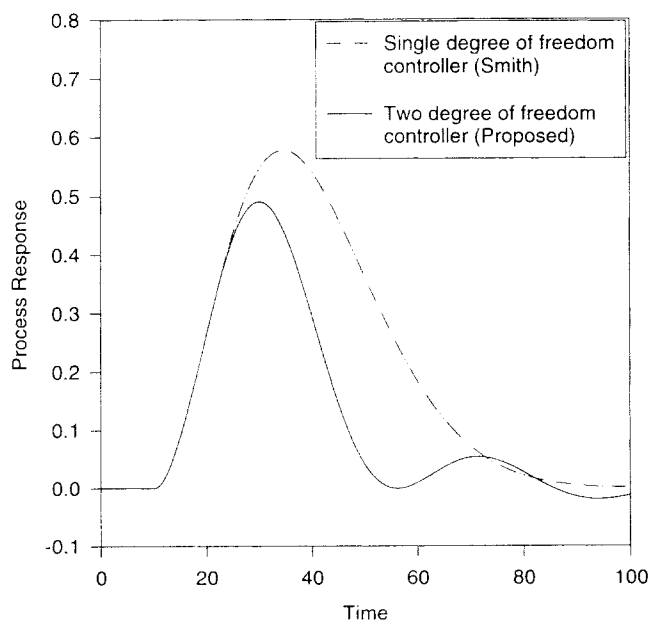


Figure 11. Disturbance rejection responses by single degree of freedom controller with $\lambda=10$ and two degree of freedom controller with $\lambda=\lambda_r=5$ for $G(s)=e^{-10s}/(10s+1)^2$.

$$G_c = \frac{qq_d(s)}{1 - G(s)qq_d(s)} = \frac{(\tau^2 s^2 + 2\zeta\tau s + 1)(\alpha_2 s^2 + \alpha_1 s + 1)}{K[(\lambda s + 1)^4 - (\alpha_2 s^2 + \alpha_1 s + 1)e^{-\theta s}]} \quad (36)$$

The PID parameters can be evaluated from Eqs. 7, 8 and 9.

The poles of $G_D(s)$ are

$$p_1, p_2 = \frac{-\zeta\tau \pm \sqrt{(\zeta^2 - 1)\tau^2}}{\tau^2} \quad (37)$$

Using the poles given by Eq. 37, the parameters α_i are obtained by solving the following equations, given a value for the filter time constant λ . λ is most easily found by trial and error using the IMCTUNE software or the 2DFPID macro described previously

$$[1 - G(s)qq_d(s)]|_{s=p_1, p_2} = 0 \quad (38)$$

$$\left[1 - \frac{(\alpha_2 s^2 + \alpha_1 s + 1)}{(\lambda s + 1)^4} e^{-\theta s} \right] \Big|_{s=p_1, p_2} = 0 \quad (39)$$

Figure 11 compares the closed-loop disturbance response from a single degree of freedom controller tuned by Smith's (1975) method with the above two-degree of freedom controller for the process $e^{-10s}/(10s + 1)^2$. The controller for this process with $\lambda = \lambda_r = 5$ is given by $K_c = 1.84$, $\tau_I = 21.36$, $\tau_D = 6.45$; $q_r = (5s + 1)^2/(86.2s^2 + 18.39s + 1)$.

Conclusions

We generalize the IMC-PID approach and show how to obtain PID parameters for general process models. The PID controller is obtained by taking the first three terms of the Maclaurin series expansion of the single-loop form of the IMC controller. In frequency and time domain, approximation of the ideal controller by the Maclaurin series approaches the ideal controller more accurately than that of existing methods. The PID controllers tuned by the proposed method give better closed-loop responses than those tuned by other tun-

ing methods. A new design method of two degree of freedom controllers was also proposed in this article. Such controllers also provided significantly improved dynamic performance over single degree of freedom controllers when the disturbances entered through the process.

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Notation

C = controlled variable
 F = error
 M = manipulated variable
 K = process gain
 R = set point
 ξ = damping ratio, dimensionless
 τ_D = derivative tuning parameter
 ISE = integral of the square error
 $ITAE$ = integral of the time-weighted absolute error

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